1.1 $f_a'(x) = \sqrt{a-x} - \frac{1}{2}x \cdot \frac{1}{\sqrt{a-x}} = \frac{2a-3x}{2\sqrt{a-x}}$

$f_a''(x) = \frac{-6\sqrt{a-x} + (2a-3x)\cdot\frac{1}{\sqrt{a-x}}}{4(a-x)} = \frac{-4a+3x}{4(a-x)\sqrt{a-x}}$

1.2

1.2.1 $D_a = \{x \mid x < a\}$

1.2.2 Schnittpunkte mit der x-Achse: $N_1(0|0)$; $N_2(a|0)$

Extrempunkte: $f_a'(x_E) = 0 \curvearrowright 2a-3x_E = 0 \curvearrowright x_E = \frac{2}{3}a$

$f_a''(x_E) = -\frac{3}{2\cdot\sqrt{\frac{2}{3}a}} < 0 \curvearrowright H_a\left(\frac{2}{3}a \middle| \frac{2}{3}a\sqrt{\frac{1}{3}a}\right)$

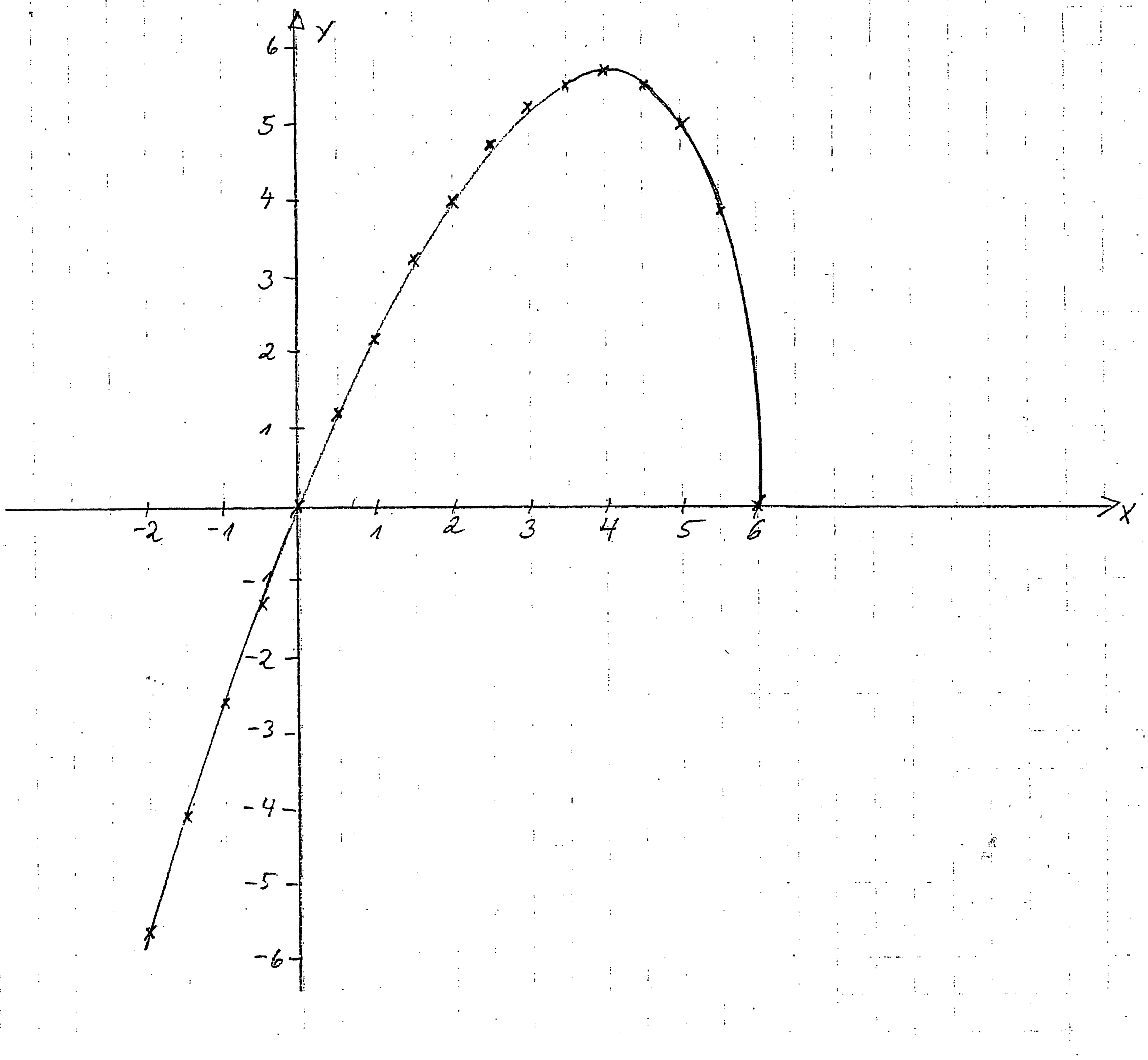
1.2.3 Wendepunkte: $f_a''(x) = 0 \curvearrowright -4a+3x = 0 \curvearrowright x = \frac{4}{3}a \notin D$

Es existieren keine Wendepunkte.

1.2.4 Asymptoten: K_a hat keine Asymptoten.

1.2.5 Schaubild:

-2	-1,5	-1	-0,5	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6
-5,7	-4,1	-2,6	-1,3	0	1,2	2,2	3,2	4	4,7	5,2	5,5	5,7	5,5	5	3,9	0



1.3 Ortslinie: $x = \frac{2}{3}a \curvearrowright a = \frac{3}{2}x$

$y = \frac{2}{9}a\sqrt{3a} = \frac{2}{9}\cdot\frac{3}{2}x\cdot\sqrt{\frac{9}{2}x} = x\cdot\sqrt{\frac{x}{2}}$

1.4

1.4.1. A_{Ka}: $A_{Ka} = \int_0^a x\sqrt{a-x}\,dx$

$u(x) = x \qquad v'(x) = \sqrt{a-x}$
$u'(x) = 1 \qquad v(x) = -\frac{2}{3}(a-x)^{\frac{3}{2}}$

$A_{Ka} = \left[-\frac{2}{3}x(a-x)^{\frac{3}{2}}\right]_0^a + \frac{2}{3}\int_0^a (a-x)^{\frac{3}{2}}\,dx$

$A_{Ka} = \left[-\frac{2}{3}\cdot\frac{2}{5}(a-x)^{\frac{5}{2}}\right]_0^a = \frac{2}{3}\cdot\frac{2}{5}a^{\frac{5}{2}} = \frac{4}{15}\sqrt{a^5}$

A_Δ: $A_\Delta = \frac{1}{2}a\cdot\frac{2a}{9}\sqrt{3a} = \frac{1}{9}a^2\sqrt{3a} = \frac{1}{9}\sqrt{3a^5}$

$\frac{A_{Ka}}{A_\Delta} = \frac{4\cdot 9}{15\cdot\sqrt{3}} = \frac{4}{5}\sqrt{3}$

1.4.2 $A_{K6} = 12\cdot\sqrt{2}$

1.5.

1.5.1 1) $C(2/4)$

2) Normale in C zu K_6: $f_6'(2) = 1{,}5 \curvearrowright m_N = -\frac{2}{3}$

n_C: $y = -\frac{2}{3}x + b \qquad b = 4 + \frac{4}{3} = \frac{16}{3} \qquad y = -\frac{2}{3}x + \frac{16}{3}$

3) M: $\{M\} = n_C \cap x$-Achse $\quad 0 = -\frac{2}{3}x_M + \frac{16}{3} \curvearrowright x_M = 8$

$M(8/0)$

4) Radius: $r = \sqrt{(8-2)^2+4^2} = \sqrt{52} = 2\sqrt{13}$

1.5.2. zu zeigen $MC \perp f_6'(2)$:

$f_6'(2) = \frac{3}{2} \qquad m_{MC} = \frac{0-4}{8-2} = -\frac{2}{3}$

$f_6'(2)\cdot m_{MC} = -1$ qed.

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2.1 $f'(x) = (2x^2 + \frac{1}{2}x - \frac{3}{4}) \cdot e^{2x}$

$f''(x) = (4x^2 + 5x - 1) \cdot e^{2x}$

2.2.1 $x^2 - \frac{3}{4}x = 0 \iff x(x - \frac{3}{4}) = 0 \iff x_1 = 0,\ x_2 = \frac{3}{4}$

2.2.2 $f(-x) \neq f(x),\ f(-x) \neq -f(x) \Rightarrow f$ ist nicht symm.

2.2.3 Im Endlichen keine Asymptoten!

$x \to +\infty: \ y = x^2 e^{2x} \quad x \to -\infty: \ y = 0$

2.2.4 $f'(x) \overset{!}{=} 0: \ 2x^2 + \frac{1}{2}x - \frac{3}{4} \overset{!}{=} 0$

$\Rightarrow x_{3/4} = -\frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{3}{8}}$

$\Rightarrow x_3 = \frac{1}{2}; \ x_4 = -\frac{3}{4}$

$f''(x_3) = (4 \cdot \frac{1}{4} + \frac{5}{2} - 1)e^1 > 0$ d.h. MIN

$f''(x_4) = (4 \cdot \frac{9}{16} - \frac{15}{4} - 1)e^{-3/2} < 0$ d.h. MAX

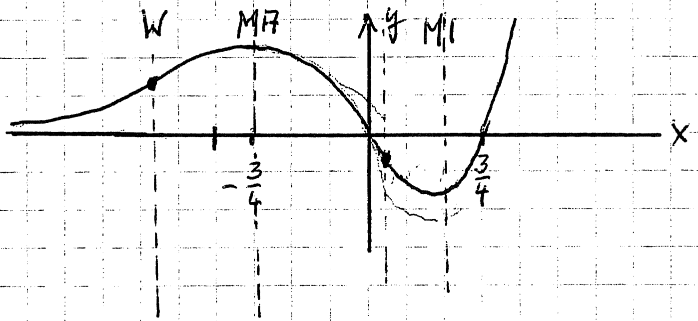
MINIMUM: $(\frac{1}{2} | -\frac{1}{8}e)$; MAXIMUM: $(-\frac{3}{4} | \frac{9}{8}e^{-3/2})$

2.2.5 $4x^2 + 5x - 1 \overset{!}{=} 0 \Rightarrow x_{5/6} = -\frac{5}{8} \pm \sqrt{\frac{25}{64} + \frac{1}{4}}$

$\Rightarrow x_5 = -\frac{5}{8} + \frac{1}{8}\sqrt{41} \approx 0{,}175$

$x_6 = -\frac{5}{8} - \frac{1}{8}\sqrt{41} \approx -1{,}425$

2.2.6



2.3 $\int (x^2 - \frac{3}{4}x)e^{2x}dx = (x^2 - \frac{3}{4}x)\frac{1}{2}e^{2x} - \int (2x - \frac{3}{4})\frac{1}{2}e^{2x}dx$

$= \frac{1}{2}(x^2 - \frac{3}{4}x)e^{2x} - \frac{1}{2}\left[(2x - \frac{3}{4}) \cdot \frac{1}{2}e^{2x} - \int 2 \cdot \frac{1}{2}e^{2x}dx\right]$

$= (\frac{1}{2}x^2 - \frac{3}{8}x)e^{2x} + (-\frac{1}{2}x + \frac{3}{16})e^{2x} + \frac{1}{4}e^{2x} + C$

$= (\frac{1}{2}x^2 - \frac{7}{8}x + \frac{7}{16})e^{2x} + C$

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2.4 Ansatz: $F(x)=(ax^2+bx+c)\cdot e^{2x}$

$\Rightarrow F'(x)=(2ax^2+(2a+2b)x+(b+2c))\cdot e^{2x}$

mit $F'(x)=(x^2-\frac{3}{4}x)e^{2x}$ folgt hieraus:

$2a=1$ also $a=\frac{1}{2}$; $2a+2b=-\frac{3}{4}$ also $b=-\frac{7}{8}$

$b+2c=0$ also $c=\frac{7}{16}$

2.5 $\int_0^{3/4}(x^2-\frac{3}{4}x)e^{2x}dx=(\frac{1}{2}x^2-\frac{7}{8}x+\frac{7}{16})e^{2x}\Big|_0^{3/4}$

$=(\frac{9}{32}-\frac{21}{32}+\frac{7}{16})e^{3/2}-\frac{7}{16}$

$=\frac{1}{16}e^{3/2}-\frac{7}{16}$

$=-0{,}158$

$\Rightarrow F_f=0{,}158$

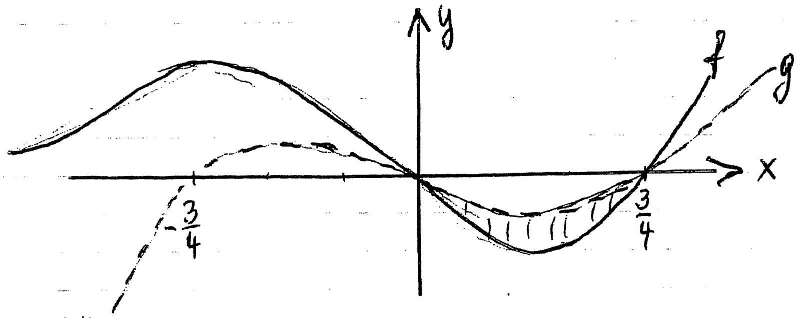
2.6. $A(u)=\int_u^0(x^2-\frac{3}{4}x)e^{2x}dx=(\frac{1}{2}x^2-\frac{7}{8}x+\frac{7}{16})e^{2x}\Big|_u^0$

$\Rightarrow A(u)=\frac{7}{16}-(\frac{1}{2}u^2-\frac{7}{8}u+\frac{7}{16})e^{2u}$

2.7 $\int_{-\infty}^0(x^2-\frac{3}{4}x)e^{2x}dx=\lim_{u\to-\infty}A(u)=\frac{7}{16}$

2.8 $f(x)=g(x)\Rightarrow x(x-\frac{3}{4})e^{2x}=x(x-\frac{3}{4})\cdot(x+\frac{3}{4})$

$\Rightarrow x=0$ und $x=\frac{3}{4}$



$\int_0^{3/4}(x^3-\frac{9}{16}x)dx=(\frac{1}{4}x^4-\frac{9}{32}x^2)\Big|_0^{3/4}$

$=\frac{81}{1024}-\frac{81}{512}=-\frac{81}{1024}=-0{,}079$

$\Rightarrow$ mit 2.5.

$F=F_f-F_g=0{,}079$

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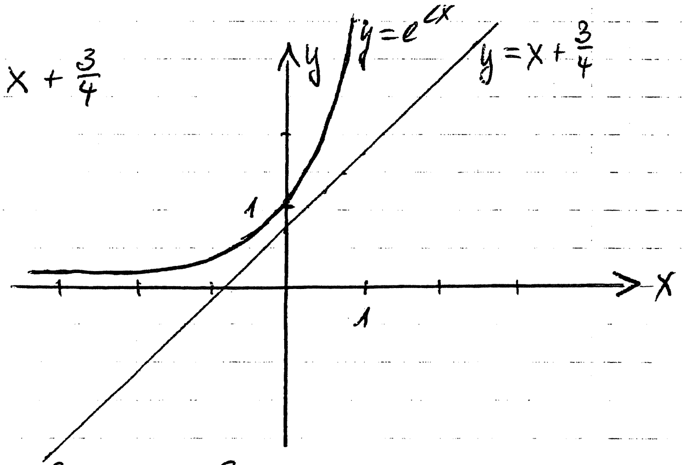
2.9.1 $e^{2x} = x + \frac{3}{4}$



Beh.: $e^{2x} > x + \frac{3}{4}$ für $x \in [0, \frac{3}{4}]$

Bew.: $x = 0$: $e^{2 \cdot 0} = 1 > \frac{3}{4} = 0 + \frac{3}{4}$

Vergl. der Steigungen:

$$(e^{2x})' = 2 \cdot e^{2x} \geq 2 \cdot e^{0} = 2 \quad x \geq 0$$

$$(x + \tfrac{3}{4})' = 1$$

$\Rightarrow$ keine weiteren Lösungen in $[0, \infty)$

2.9.2 Beh.: $e^{2x} > x + \frac{3}{4} \quad \forall x \in \mathbb{R}$

Bew.: $e^{2x} \gg x + \frac{3}{4}$ für $x \to +\infty$

$e^{2x} \gg x + \frac{3}{4}$ für $x \to -\infty$

Untersuchung an der Stelle (den Stellen) mit gleicher Steigung:

$$(e^{2x})' = (x + \tfrac{3}{4})'$$

$$\Rightarrow 2 \cdot e^{2x} = 1$$

$$\Rightarrow x_0 = -\tfrac{1}{2} \ln 2 \quad (= -0{,}347)$$

$$e^{2x_0} = e^{-\ln 2} = \tfrac{1}{2} > -\tfrac{1}{2}\ln 2 + \tfrac{3}{4} = 0{,}403$$

q.e.d.

Abiturprüfung 2004
Aufgabe 3

3.1 $E_1: \quad ax_1 + bx_2 + cx_3 = d$

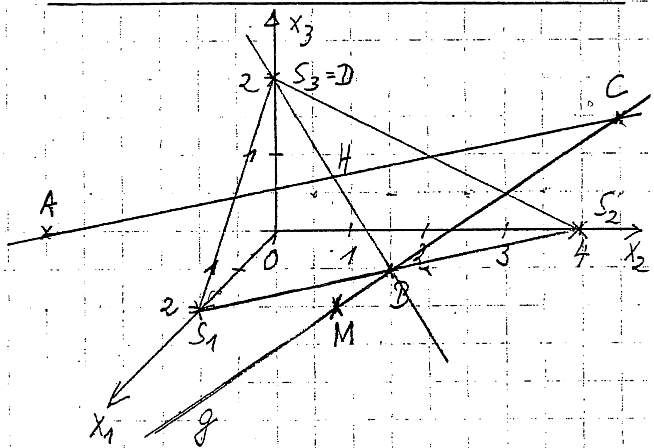
$$\begin{array}{l} 2a - 2b + c - d = 0 \\ 1a + 2b - d = 0 \\ -a + 4b + c - d = 0 \\ \hline 2a - 2b + c - d = 0 \\ -6b + c + d = 0 \\ 6b + 3c - 3d = 0 \\ \hline 2a - 2b + c - d = 0 \\ -6b + c + d = 0 \\ 4c - 2d = 0 \\ \hline \end{array}$$

Wähle $d = 2 \curvearrowright c = 1 \curvearrowright b = \frac{1}{2} \curvearrowright a = 1$

$\underline{\underline{E_1: \; 2x_1 + x_2 + 2x_3 = 4}}$

Schnittpunkte mit den Koordinatenachsen

$S_1(2/0/0)$; $S_2(0/4/0)$; $S_3(0/0/2)$



Bestimme t so, dass $D_t \in E_1$

$2(9+3t) + 3 + t + 16 + 4t = 4$

$\curvearrowright 33 + 11t = 0 \curvearrowright \underline{t = -3}$

$\curvearrowright \underline{\underline{D(0/0/2)}}$

H; $\overline{AC}$: $\vec{x} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} + s \begin{pmatrix} -3 \\ 6 \\ 0 \end{pmatrix}$

$\overline{BD}$: $\vec{x} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + r \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$\overline{AC} \cap \overline{BD} = \{H\}$

$$\left.\begin{array}{l} 2 - 3s = r \\ -2 + 6s = 2r \\ 1 = 2 - 2r \end{array}\right\} \; r = \frac{1}{2}; \; s = \frac{1}{2}; \; \curvearrowright \underline{\underline{H\left(\tfrac{1}{2}/1/1\right)}}$$

3.2.1 g:

$$\begin{array}{l} 2x_1 + x_2 + 2x_3 = 4 \\ x_1 + 2x_2 - 2x_3 = 5 \\ \hline 2x_1 + x_2 + 2x_3 = 4 \\ -3x_2 + 6x_3 = -6 \end{array}$$

$x_3 = \lambda$; $x_2 = 2 + 2\lambda$; $x_1 = 1 - 2\lambda$

$\curvearrowright g: \; \vec{x} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$

Schnittwinkel:

x_2x_3-Ebene: $\vec{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\curvearrowright \cos \alpha = \dfrac{\vec{n} \cdot \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}}{|\vec{n}| \left|\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}\right|} = -\dfrac{2}{3}$

$\curvearrowright \underline{\underline{\alpha = 131{,}8^\circ}} \quad \curvearrowright \underline{\underline{\alpha' = 48{,}2^\circ}}$

3.2.2 M^* ist Mittelpunkt von Strecke $\overline{AC}$: $\vec{m}^* = \vec{a} + \frac{1}{2}\overline{AC} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -3 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0{,}5 \\ 1 \\ 1 \end{pmatrix}$

E^* ist Ebene mit $M^* \in E^* \wedge \overline{AC} \perp E^*$: $-3x_1 + 6x_2 = d \curvearrowright d = -3 \cdot \frac{1}{2} + 6 = 4{,}5$

$\{M\} = g \cap E^*$:

$$\left.\begin{array}{l} E^*: \; -3x_1 + 6x_2 = 4{,}5 \\ g: \; \vec{x} = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} \end{array}\right\} \begin{array}{rl} -3(-1+2s) + 6(4-2s) &= 4{,}5 \\ \curvearrowright 3 - 6s + 24 - 12s &= 4{,}5 \\ 27 - 18s &= 4{,}5 \\ -18s &= -22{,}5 \\ s &= 1{,}25 \end{array}$$

$\curvearrowright \underline{\underline{M(1{,}5/1{,}5/-0{,}25)}}$

3.2.3 E' ist Ebene mit $T \in E' \wedge \overline{AC} \perp E'$

E': $\vec{n} = \overrightarrow{AC} = \begin{pmatrix} -3 \\ 6 \\ 0 \end{pmatrix}$ $\quad -3x_1 + 6x_2 = d$

$T \in E' \leadsto -4{,}5 + 9 = d \leadsto d = 4{,}5$

E': $-3x_1 + 6x_2 = 4{,}5 \qquad \overline{AC}: \vec{x} = \begin{pmatrix} 2 \\ -2 \\ +1 \end{pmatrix} + s \begin{pmatrix} -3 \\ 6 \\ 0 \end{pmatrix}$

Lotfußpunkt F: $\{F\} = \overline{AC} \cap E'$

$-3(2-3s) + 6(-2+6s) = 4{,}5 \quad \leadsto \underline{\underline{F(0{,}5|1|1)}}$

$-6 + 9s - 12 + 36s = 4{,}5$

$45s = 22{,}5$

$s = \frac{1}{2}$

Abstand $|\overline{TF}| = \sqrt{1^2 + 0{,}5^2 + 1{,}25^2} = \underline{\underline{1{,}68}}$

3.3.1 Spiegeln von A an E_2:

1. Bestimme Normale n_A zu E_2 durch A: $n_A: \vec{x} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

2. Bestimme F: $\{F\} = E_2 \cap n_A$

$2 + s + 2(-2+2s) - 2(1-2s) - 5 = 0$

$2 + s - 4 + 4s - 2 + 4s - 5 = 0$

$-9 + 9s = 0 \leadsto s = 1 \leadsto F(3|0|-1)$

3. Bestimme Spiegelpunkt A': A': $\vec{a}' = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} + 2\overrightarrow{AF} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$

$\underline{\underline{A'(4|2|-3)}}$

3.3.2 $\{S\} = n_A \cap g$

$2 + r = -1 + 2s \qquad r - 2s = -3$

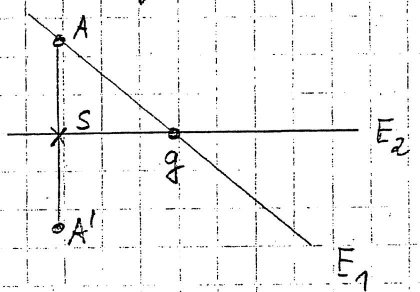
$-2 + 2r = 4 - 2s \qquad 2r + 2s = 6 \qquad \leadsto r - 4r = -3 \leadsto r = 1$

$1 - 2r = 1 - s \qquad -2r + s = 0$

$\underline{\underline{\leadsto S(3|0|-1)}}$

Die beiden Ebenen stehen senkrecht aufeinander

Begründung: Wenn $E_1 \not\perp E_2$, dann ist nach Skizze $S \notin g$



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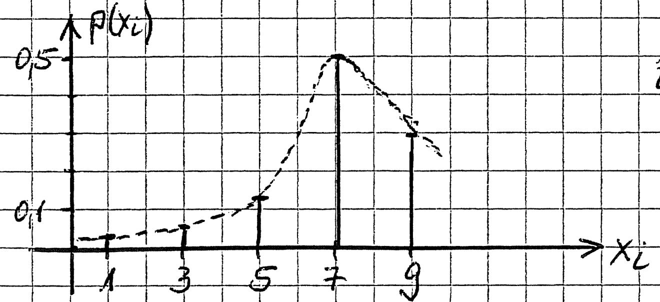
4.1

X	1	3	5	7	9
$p(x_i)$	$1/40$	$1/20$	$1/8$	$1/2$	$3/10$



$\frac{1}{40} = 0{,}025$

4.2 $E(X) = 1\cdot\frac{1}{40} + 3\cdot\frac{1}{20} + 5\cdot\frac{1}{8} + 7\cdot\frac{1}{2} + 9\cdot\frac{3}{10} = \frac{280}{40} = 7$

$V(X) = (7-1)^2\cdot\frac{1}{40} + (7-3)^2\cdot\frac{1}{20} + (7-5)^2\cdot\frac{1}{8} + 0 + (7-9)^2\cdot\frac{3}{10}$

$= \frac{136}{40} = 3{,}4$

$\sigma(X) = 1{,}84$

Mit geringer Streuung wird die Häufigkeit der Zahl 7 im Mittel groß sein.

4.3 4.3.1 $p(E) = \frac{1}{40} + \frac{3}{10} = \frac{13}{40}$ $p(F) = 1 - p(E) = \frac{27}{40}$

$p(E \cap F) = 0$

4.3.2 $p(E \cap F) \neq p(E)\cdot p(F) \Rightarrow E, F$ abhängig

4.4 $p(F) = \frac{1}{40} + \frac{1}{20} + \frac{1}{8} = \frac{8}{40} = \frac{1}{5}$

4.5 4.5.1 $p(X=10) = \binom{10}{10}\cdot\left(\frac{1}{5}\right)^{10}\cdot\left(\frac{4}{5}\right)^{0} = \left(\frac{1}{5}\right)^{10}$

4.5.2 $p(X \geq 1) = 1 - p(X=0)$

$= 1 - \binom{n}{0}\cdot\left(\frac{1}{5}\right)^0\cdot\left(\frac{4}{5}\right)^n \overset{!}{\geq} 0{,}4$

$\Rightarrow 1 - \left(\frac{4}{5}\right)^n \geq 0{,}4$

$\Rightarrow \left(\frac{4}{5}\right)^n \leq 0{,}6$

$\Rightarrow 0{,}8^n \leq 0{,}6$

$\Rightarrow$ ab $n = 3$ erfüllt

Das Steuer muss mindestens 3x gedreht werden.

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4.5.3 $p(21 \le X \le 35) = F_{100;0,2}(35) - F_{100;0,2}(20)$

$= 0,9999 - 0,5595 = 0,4404$

4.6 $E = n \cdot p = 100 \cdot \frac{1}{5} = 20$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{100 \cdot \frac{1}{5} \cdot \frac{4}{5}} = 4$

$\Rightarrow p(\mu - 2\sigma \le X \le \mu + 2\sigma) \approx 0,95$

$\Rightarrow$ Bereich: $[12, 28]$

4.7 $H_0: p_0 = \frac{3}{5}$ d.h. $\mu = 50 \cdot \frac{3}{5} = 30$

$\sigma = \sqrt{50 \cdot \frac{3}{5} \cdot \frac{2}{5}} = 2\sqrt{3} = 3,46$

Signifikanzniveau 5% $\Rightarrow$ Faktor ist 1,96

$\Rightarrow [\mu - c\sigma; \mu + c\sigma] = [23,21; 36,78]$

$\Rightarrow$ Annahmebereich: $[24, 36]$

Da $20 \notin [24, 36]$ wird die Hypothese abgelehnt

4.8 $h = \frac{250}{1000} = \frac{1}{4}$

4.8.1 $I = \left[h - 1,96\sqrt{\frac{h(1-h)}{n}}; h + 1,96\sqrt{\frac{h(1-h)}{n}}\right]$

$= [0,22316; 0,2768]$

4.8.2 $d = 0,0536$

4.8.3 $n \ge \frac{1,96^2}{d^2} \Rightarrow n \ge \frac{1,96^2}{(0,01)^2} = 38416$

$\Rightarrow$ man müsste 38416 mal drehen

4.9 $\frac{1}{40}$ $\frac{1}{20}$ $\frac{1}{8}$ a priori

1 3 5

$\frac{3}{400} + \frac{1}{40} + \frac{1}{64} = \frac{77}{1600}$

$1 \xrightarrow{\frac{3}{10}} 10$, $3 \xrightarrow{\frac{1}{2}} 10$, $5 \xrightarrow{\frac{1}{8}} 10$

$\frac{3}{400}$ $\frac{1}{40}$ $\frac{1}{64}$ a posteriori

$\Rightarrow (1/9) \rightarrow \frac{3/400}{77/1600} = \frac{12}{77} = 0,156$

$(3/7) \rightarrow \frac{1/40}{77/1600} = \frac{40}{77} = 0,519$ $\leftarrow$ Antwort

$(5/5) \rightarrow \frac{1/64}{77/1600} = \frac{25}{77} = 0,325$

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4.10

Summe	Ereignisse	Wahrsch.keiten
2	(1,1)	0,000625
4	(3,1),(1,3)	0,0025
6	(5,1),(3,3),(1,5)	0,0087
8	(7,1),(5,3),(3,5),(1,7)	0,0375
10	(9,1),(7,3),(5,5),(3,7),(1,9)	0,080625
12	(9,3),(7,5),(5,7),(3,9)	0,155
14	(9,5),(7,7),(5,9)	0,325
16	(9,7),(7,9)	0,3
18	(9,9)	0,09